

# A Complete Equation of State for Astrophysical Simulations

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**Abstract** We construct a complete equation of state (EOS) covering a wide range of temperature, proton fraction, and baryon density for the use of astrophysical simulations. We employ the relativistic mean-field (RMF) theory to describe nuclear interactions and adopt the Thomas-Fermi approximation to describe the nonuniform nuclear matter. The uniform matter and nonuniform matter are studied consistently using the same RMF theory.

**Key words** equation of state—Thomas-Fermi approximation

## 1. INTRODUCTION

The equation of state (EOS) is an important input in various astrophysical studies like neutron stars and supernovae<sup>[1]</sup>. In past decades, many efforts have been devoted to construct the EOS for supernova simulations<sup>[2,3,4]</sup>. There are two standard EOS's, which are commonly used in supernova simulations, namely the one by Lattimer and Swesty<sup>[2]</sup> and the one by Shen et al.<sup>[3]</sup>. The Lattimer-Swesty EOS was made by using a compressible liquid-drop model with Skyrme forces. The Shen EOS was calculated with the relativistic mean-field (RMF) model and the Thomas-Fermi approximation. In our previous work<sup>[3]</sup>, we constructed the EOS table covering a wide range of temperature  $T$ , proton fraction  $Y_p$ , and baryon mass density  $\rho_B$  for the use of supernova simulations. Recently, we have recalculated the Shen EOS with an improved design of ranges and grids according to the requirements of the EOS users, and furthermore, the presence of hyperons has also been considered<sup>[5]</sup>.

## 2. METHOD

We employ the RMF theory to describe nuclear matter with uniform or nonuniform distributions. The RMF theory has been successfully used to study various phenomena in nuclear physics. We adopt the TM1 parameter set, which can provide a good description

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of nuclear matter and finite nuclei<sup>[6]</sup>. The Lagrangian density in the RMF theory with the TM1 parameter set is given by

$$\mathcal{L}_{\text{RMF}} = \bar{\psi} [i\gamma_\mu \partial^\mu - M - g_\sigma \sigma - g_\omega \gamma_\mu \omega^\mu - g_\rho \gamma_\mu \tau_a \rho^{a\mu}] \psi + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 - \frac{1}{4} W_{\mu\nu} W^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{4} c_3 (\omega_\mu \omega^\mu)^2 - \frac{1}{4} R_{\mu\nu}^a R^{a\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu^a \rho^{a\mu}, \quad (1)$$

where  $W^{\mu\nu}$  and  $R^{a\mu\nu}$  are the antisymmetric field tensors for  $\omega^\mu$  and  $\rho^{a\mu}$ , respectively. In the RMF approach, meson fields are treated as classical fields and the field operators are replaced by their expectation values. Starting from the Lagrangian density, we can derive the equations of motion for nucleons and mesons, which are then solved self-consistently.

We study properties of dense matter with both uniform and nonuniform distributions. For uniform matter, the RMF theory can be easily used to calculate properties of nuclear matter. For nonuniform matter, we assume each heavy nucleus is located at the center of a charge-neutral Wigner–Seitz cell consisting of a vapor of nucleons, electrons, and alpha-particles. The nucleon distribution in the Wigner–Seitz cell,  $n_i(r)$  ( $i = p$  or  $n$ ), is assumed to have the form

$$n_i(r) = \begin{cases} (n_i^{\text{in}} - n_i^{\text{out}}) \left[ 1 - \left( \frac{r}{R_i} \right)^{t_i} \right]^3 + n_i^{\text{out}}, & 0 \leq r \leq R_i, \\ n_i^{\text{out}}, & R_i \leq r \leq R_{\text{cell}}, \end{cases} \quad (2)$$

where  $R_i$  and  $R_{\text{cell}}$  represent the radii of the nucleus and the Wigner–Seitz cell, respectively. The density parameters  $n_i^{\text{in}}$  and  $n_i^{\text{out}}$  are the densities at  $r = 0$  and  $r \geq R_i$ , while  $t_i$  determines the relative surface thickness of the nucleus. The distribution of alpha-particles is assumed to have a similar form, which decreases as  $r$  approaches the center of the nucleus. For a system with fixed temperature  $T$ , proton fraction  $Y_p$ , and baryon mass density  $\rho_B$ , the thermodynamically favorable state is determined by minimizing the free energy density with respect to these parameters. The results of the RMF model are used in the Thomas-Fermi calculation, so the treatments of nonuniform and uniform matter are consistent.

### 3. RESULTS

At each given  $T$ ,  $Y_p$ , and  $\rho_B$ , we perform the minimization of the free energy. The thermodynamically favorable state is the one having the lowest free energy. By comparing the free energies of nonuniform matter and uniform matter, we can determine the most favorable state and examine the phase transition between nonuniform matter and uniform matter.

In Fig. 1, we show the phase diagram in the  $\rho_B$ – $T$  plane for  $Y_p = 0.3$ . The shaded region corresponds to the nonuniform matter phase, in which heavy nuclei are formed to lower the free energy of the system. The dashed line denotes the boundary where the alpha-particle fraction  $X_\alpha$  changes between  $X_\alpha < 10^{-4}$  and  $X_\alpha > 10^{-4}$ . It is evident that heavy nuclei only exist in the medium-density and low-temperature region.

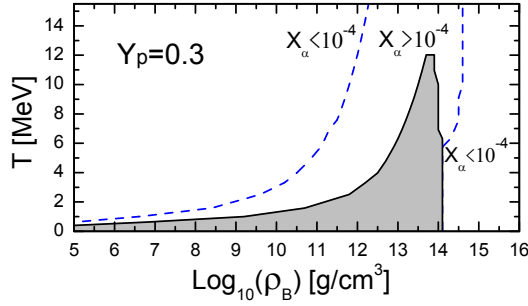


Fig. 1 Phase diagram of nuclear matter in the  $\rho_B$ - $T$  plane for  $Y_p = 0.3$ .

In Table 1, we compare the improved EOS's, namely EOS2 and EOS3<sup>[5]</sup>, with the previous EOS1<sup>[3]</sup>. The difference between EOS2 and EOS3 is that only the nucleon degree of freedom is considered in EOS2, while EOS3 includes additional contributions from  $\Lambda$  hyperons. In comparison with EOS1, several improvements have been made in EOS2 and EOS3 according to the requirements of the users. The grid spacing for temperature  $T$  is significantly reduced and a linear  $Y_p$  grid is adopted instead of the logarithmic  $Y_p$  grid used in EOS1. These improvements are very important for numerical simulations of supernovae.

**Table 1 Comparison between the EOS's discussed in this paper**

		EOS1	EOS2	EOS3
Constituents		$n, p, \alpha$	$n, p, \alpha$	$n, p, \alpha, \Lambda$
$T$ (MeV)	Range	$-1.0 \leq \log_{10}(T) \leq 2.0$	$-1.0 \leq \log_{10}(T) \leq 2.6$	$-1.0 \leq \log_{10}(T) \leq 2.6$
	Grid spacing	$\Delta \log_{10}(T) \simeq 0.1$	$\Delta \log_{10}(T) = 0.04$	$\Delta \log_{10}(T) = 0.04$
$Y_p$	Range	$-2 \leq \log_{10}(Y_p) \leq -0.25$	$0 \leq Y_p \leq 0.65$	$0 \leq Y_p \leq 0.65$
	Grid spacing	$\Delta \log_{10}(Y_p) = 0.025$	$\Delta Y_p = 0.01$	$\Delta Y_p = 0.01$
$\rho_B$ (g cm <sup>-3</sup> )	Range	$5.1 \leq \log_{10}(\rho_B) \leq 15.4$	$5.1 \leq \log_{10}(\rho_B) \leq 16$	$5.1 \leq \log_{10}(\rho_B) \leq 16$
	Grid spacing	$\Delta \log_{10}(\rho_B) \simeq 0.1$	$\Delta \log_{10}(\rho_B) = 0.1$	$\Delta \log_{10}(\rho_B) = 0.1$

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